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Technical Note

A computationally efficient method for Monte Carlo simulation of diffuse radiant emission or reflection

Farshad Kowsary*

Mechanical Engineering Department, Faculty of Engineering, University of Tehran, Tehran 14399, Iran

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Perhaps the most often 'CALL'ed subroutine in any Monte Carlo computer code written to perform radiant exchange calculations in an enclosure is that which randomly determines the diffuse direction for a ray bundle leaving a surface of the enclosure. This is because surface emission (almost always) as well as reflection (very often) are characterized as being diffuse. Thus, a more efficient way of determining the diffuse direction can save considerable computational time and effort.

In the conventional method, as was presented by Howell and Perlmutter [1], the diffuse direction from point D on surface S in Fig. 1a is obtained by random selection of angles θ and ϕ with respect to the local coordinate system shown in Fig. 1a. Two random numbers $0 < R_\theta < 1$ and $0 < R_\phi < 1$ are selected, and θ and φ are then calculated using

$$\theta = \sin^{-1}\sqrt{R_\theta} \quad \text{and} \quad \phi = 2\pi R_\phi \tag{1}$$

The method suggested here is based on the fact that the diffuse radiant energy leaving a point source located on the inner surface of a sphere is distributed uniformly throughout the inner surface of the sphere. This is a corollary to a well-known property of the sphere that the diffuse view factor between two area elements on the interior surface of a sphere depends only on the size of the receiving element [2]. This property is utilized in designing integrating spheres. Looking at this idea from the opposite direction, it is clear that if a large number of points on the interior surface of the sphere are selected in a manner such that their distribution is uniform throughout the sphere, and these points are then used as means of defining paths of rays emanating from a source on the interior surface of the sphere, then the source must be a diffuse one, or the diffuse emission from the source is appropriately simulated.

By referring to Fig. 1b, it can be seen that the diffuse direction from a point D on surface S may be easily obtained by random selection of a point on the surface of an imaginary sphere which is tangent to surface S at point D. Points on the sphere are identified by azimuthal and circumferential angles, θ and ϕ, respectively, relative to a coordinate system whose origin is the center of the sphere, as shown in Fig. 1b. The size of the sphere is arbitrary. Now the question is the manner by which these angles are determined in order to have a uniform distribution of points on the interior surface of the sphere. This follows from the basic procedure for finding the *inverse probability functions* for θ and ϕ to satisfy the condition of uniform distribution of points on the interior surface of the sphere. The procedure is as follows.

An infinitesimal area element $\mathrm{d}A$ on a sphere having an arbitrary radius R is given by

$$\mathrm{d}A = R^2 \sin\theta \,\mathrm{d}\theta\,\mathrm{d}\phi$$

By dividing this equation by the surface area of the entire sphere, $4\pi R^2$, a probability density function for the distribution of area on the sphere is obtained as

$$p(\theta,\varphi) = \frac{\sin\theta}{4\pi}\mathrm{d}\theta\,\mathrm{d}\phi$$

Although $p(\theta,\phi)$ is a function of two variables, it can be written as a product of two single variable probability density functions $p_\theta(\theta)$ and $p_\phi(\phi)$. That is,

$$p(\theta,\phi) = p_\theta(\theta)\cdot \mathrm{p}_\phi(\phi)$$

Now by noting that the integrals of $p_\theta(\theta)$ and $p_\phi(\phi)$ over their entire domain have to equal to unity, we, therefore, must have

* Corresponding author.

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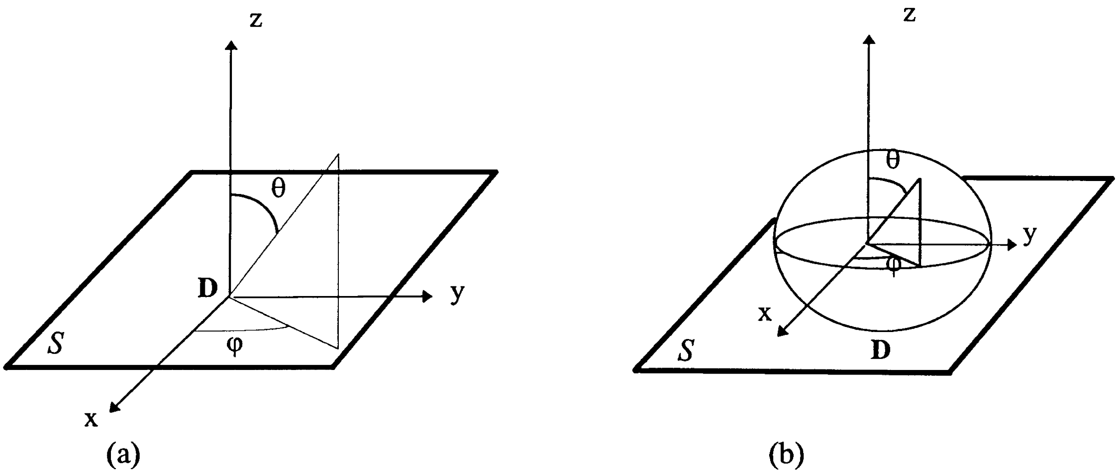

Fig. 1. Determination of the diffuse direction by: (a) the conventional method and (b) the tangent sphere method.

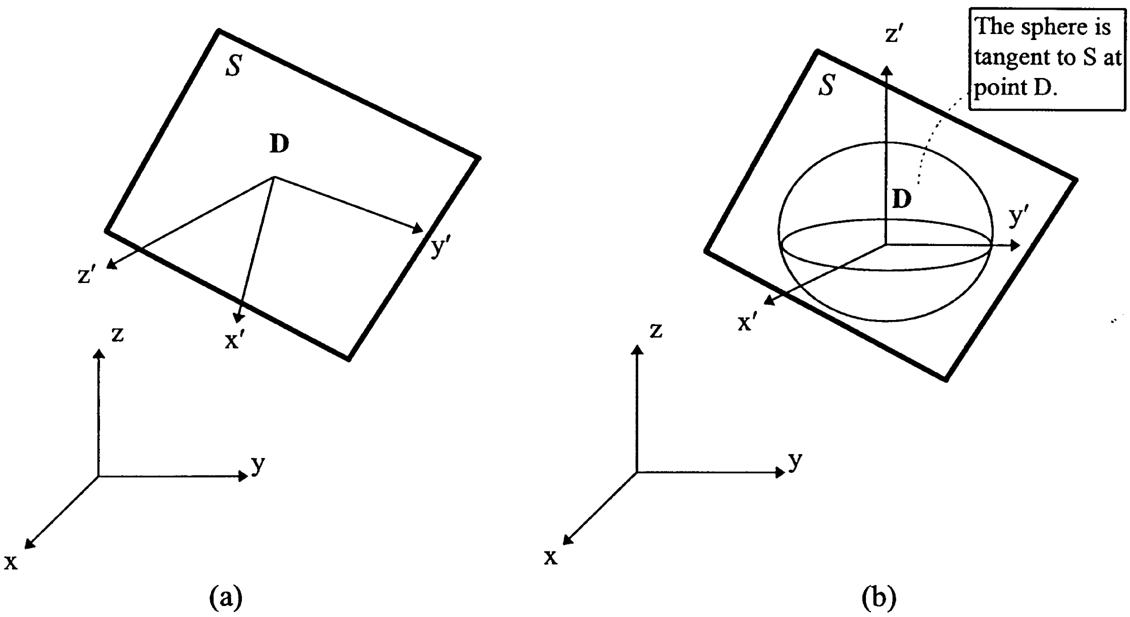

Fig. 2. Diffuse direction for a general coordinate system: (a) the conventional method (b) the tangent sphere method.

$$p_\theta(\theta) = \frac{\sin\theta}{2}\mathrm{d}\theta$$

$$p_\phi(\phi) = \frac{1}{2\pi}\mathrm{d}\phi$$

The cumulative distribution functions are then obtained as

$$P(\theta) = \int_{\theta'=0}^{\theta} p_\theta(\theta') = \frac{1}{2}(1-\cos\theta)$$

$$P(\phi) = \int_{\phi'=0}^{\phi} p_\phi(\phi')\,\mathrm{d}\phi' = \frac{\phi}{2\pi}$$

Hence the inverse probability functions are obtained by setting two random numbers R_θ and R_ϕ equal to $P(\theta)$ and $P(\phi)$, and then solving for θ and ϕ, respectively, to get

$$\theta = \cos^{-1}(2R_\theta - 1) \quad \text{and} \quad \varphi = 2\pi R_\varphi \tag{2}$$

The advantage of the 'tangent sphere' method over the conventional method becomes obvious when they both are applied to an arbitrarily oriented surface of the enclosure. As is shown in Fig. 2a, the diffuse direction given by equation (1) is determined relative to the local coordinate system $x'y'z'$ which, in general, is both rotated and translated with respect to the global coordinate system xyz defined for the enclosure. Thus, some effort is needed to convert the diffuse direction obtained relative to $x'y'z'$ into that for the global xyz coordinate system. On the other hand, using the "tangent sphere" method, one is free to choose a local coordinate system $x'y'z'$ whose axes are parallel to the global coordinate system xyz. This is due to the inherent geometrical symmetry of the sphere which does not restrict us to choose, for instance, z' axis in the direction of normal to surface S at point D. Once the diffuse direction is determined using equation (2) for the local $x'y'z'$ coordinate system, the conversion of this direction to coordinates of xyz is clearly much easier because both coordinate systems are parallel.

References

[1] Howell JR, Perlmutter M. Monte Carlo solution of thermal transfer through radiant media between gray walls. Journal of Heat Transfer 1964;2:116–22.

[2] Sparrow EM, Jonsson, VK. Absorption and emission characteristics of diffuse spherical enclosures. Technical Note D-1289, NASA, 1962.